

BEFORE THE BOARD OF PUBLIC EDUCATION
OF THE STATE OF MONTANA

In the matter of the adoption of New Rule I through New Rule XII,)	NOTICE OF PUBLIC HEARING ON
amendment of ARM 10.54.4010)	PROPOSED ADOPTION,
through 10.54.4013, 10.54.4020)	AMENDMENT, AND REPEAL
through 10.54.4023, 10.54.4030)	
through 10.54.4033, and 10.54.4040)	
through 10.54.4043, and repeal of)	
ARM 10.54.4050 through 10.54.4053,)	
10.54.4060 through 10.54.4063,)	
10.54.4070 through 10.54.4073, and)	
10.54.4087 through 10.54.4098)	
pertaining to math content standards)	
and performance descriptors)	

TO: All Concerned Persons

1. On June 22, 2009, at 10:30 a.m. the Board of Public Education will hold a public hearing in the conference room at 1300 11th Avenue, Helena, Montana, to consider the proposed adoption, amendment, and repeal of the above-stated rules.

2. The Board of Public Education will make reasonable accommodations for persons with disabilities who wish to participate in this rulemaking process or need an alternative accessible format of this notice. If you require an accommodation, contact the Board of Public Education no later than 5:00 p.m. on June 8, 2009 to advise us of the nature of the accommodation that you need. Please contact Steve Meloy, P.O. Box 200601, Helena, MT 59620-0601, telephone: (406) 444-6576, FAX: (406) 444-0847, e-mail: smeloy@mt.gov.

3. Statement of Reasonable Necessity: The Board of Public Education has determined it is reasonable and necessary to adopt, amend, and repeal rules relating to mathematics content standards and performance descriptors pursuant to ARM 10.54.2503 Standards Review Schedule. The board has determined that to stay consistent with the legislative intent of Senate Bill 152 of the 2005 Legislative Session it must review and make contemporary amendments to its standards. The Legislature recognizes the need to reassess educational needs on a cyclical basis and the board recognizes its standards represent the minimum standards. These standards are the basis upon which a quality system is built and maintained. The board strives to conform to a five year review cycle for every chapter of accreditation. The standards review process uses context information, criteria, processes, and procedures identified by the Office of Public Instruction with input from representatives of accredited schools.

The Montana content standards for mathematics are not about mandating curriculum or recommending specific courses in Montana's schools. Instead, they

are about preparing students to work and live successfully in a society that is increasingly technical, global, and multicultural. The board has set high expectations for the performance of Montana students at all levels; it is the responsibility of local communities and districts to determine the path for their students to achieve the goals set out in these content standards and performance descriptors.

Pursuant to Article X Section 1(2) of the Constitution of the state of Montana and 20-1-501 and 20-9-309(2)(c), MCA, these new rules and amended rules incorporate the distinct and unique cultural heritage of Montana American Indians.

4. The rules as proposed to be adopted provide as follows:

NEW RULE I GRADE 4 PERFORMANCE DESCRIPTORS AT THE ADVANCED LEVEL (1) A fourth-grade student at the advanced level in mathematics demonstrates superior performance. He/she gives responses that exhibit advanced understanding of the problem or situation presented. The student consistently demonstrates advanced conceptualization and makes connections within and between the mathematical and real world. The student is able to apply more than one process, use multiple representations, and determine solutions accurately. Reasoning and structure of responses are clearly communicated and justified.

(a) The student demonstrates these abilities under content standard 1 in the areas of:

- (i) whole number relationships;
- (ii) estimation and operations;
- (iii) whole number concepts;
- (iv) common fractions and decimals; and
- (v) length, time, and temperature.

(b) The student demonstrates these abilities under content standard 2 in the areas of:

- (i) representing data;
- (ii) evaluating data; and
- (iii) likelihood of events.

(c) The student demonstrates these abilities under content standard 3 in the areas of:

- (i) two-dimensional attributes;
- (ii) three-dimensional attributes;
- (iii) basic transformations;
- (iv) linear measurement; and
- (v) area and perimeter.

(d) The student demonstrates these abilities under content standard 4 in the areas of:

- (i) patterns and relations;
- (ii) symbols and expressions;
- (iii) properties of number and operation;
- (iv) equivalent expressions; and

- (v) numerical modeling with manipulatives.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE II GRADE 4 PERFORMANCE DESCRIPTORS AT THE PROFICIENT LEVEL (1) A fourth-grade student at the proficient level in mathematics demonstrates solid academic performance. He/she gives responses that exhibit clear understanding of the problem or situation presented and is able to make connections within the mathematical and/or real world. The student can determine a process and sufficiently communicate correct problem solving strategies through relevant representations. Although there may be occasional errors, these do not interfere with appropriate strategies. Solutions are reasonable and well supported.

(a) The student demonstrates these abilities under content standard 1 in the areas of:

- (i) whole number relationships;
- (ii) estimation and operations;
- (iii) whole number concepts;
- (iv) common fractions and decimals; and
- (v) length, time, and temperature.

(b) The student demonstrates these abilities under content standard 2 in the areas of:

- (i) representing data;
- (ii) evaluating data; and
- (iii) likelihood of events.

(c) The student demonstrates these abilities under content standard 3 in the areas of:

- (i) two-dimensional attributes;
- (ii) three-dimensional attributes;
- (iii) basic transformations;
- (iv) linear measurement; and
- (v) area and perimeter.

(d) The student demonstrates these abilities under content standard 4 in the areas of:

- (i) patterns and relations;
- (ii) symbols and expressions;
- (iii) properties of number and operation;
- (iv) equivalent expressions; and
- (v) numerical modeling with manipulatives.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE III GRADE 4 PERFORMANCE DESCRIPTORS AT THE NEARING PROFICIENCY LEVEL (1) A fourth-grade student at the nearing proficiency level in mathematics demonstrates partial mastery of the prerequisite

knowledge and skills fundamental for proficiency. He/she gives responses that exhibit some understanding of the problem or situation presented and can make rudimentary connections between the mathematical and/or real world. The student struggles to communicate effectively and uses limited evidence of representations to show understanding.

(a) The student demonstrates these abilities under content standard 1 in the areas of:

- (i) whole number relationships;
- (ii) estimation and operations;
- (iii) whole number concepts;
- (iv) common fractions and decimals; and
- (v) length, time, and temperature.

(b) The student demonstrates these abilities under content standard 2 in the areas of:

- (i) representing data;
- (ii) evaluating data; and
- (iii) likelihood of events.

(c) The student demonstrates these abilities under content standard 3 in the areas of:

- (i) two-dimensional attributes;
- (ii) three-dimensional attributes;
- (iii) basic transformations;
- (iv) linear measurement; and
- (v) area and perimeter.

(d) The student demonstrates these abilities under content standard 4 in the areas of:

- (i) patterns and relations;
- (ii) symbols and expressions;
- (iii) properties of number and operation;
- (iv) equivalent expressions; and
- (v) numerical modeling with manipulatives.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE IV GRADE 4 PERFORMANCE DESCRIPTORS AT THE NOVICE LEVEL

(1) A fourth-grade student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental. He/she gives responses that exhibit significant difficulty in understanding basic concepts, reasoning, implementing problem solving strategies, and making connections. Basic skills, representations, structure, and process development are severely lacking even though the student may attempt to solve problems. Substantial errors and lack of communication skills hinder the student's progress.

(a) The student demonstrates these abilities under content standard 1 in the areas of:

- (i) whole number relationships;
- (ii) estimation and operations;

- (iii) whole number concepts;
- (iv) common fractions and decimals; and
- (v) length, time, and temperature.
- (b) The student demonstrates these abilities under content standard 2 in the areas of:
 - (i) representing data;
 - (ii) evaluating data; and
 - (iii) likelihood of events.
- (c) The student demonstrates these abilities under content standard 3 in the areas of:
 - (i) two-dimensional attributes;
 - (ii) three-dimensional attributes;
 - (iii) basic transformations;
 - (iv) linear measurement; and
 - (v) area and perimeter.
- (d) The student demonstrates these abilities under content standard 4 in the areas of:
 - (i) patterns and relations;
 - (ii) symbols and expressions;
 - (iii) properties of number and operation;
 - (iv) equivalent expressions; and
 - (v) numerical modeling with manipulatives.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE V GRADE 8 PERFORMANCE DESCRIPTORS AT THE ADVANCED LEVEL (1) An eighth-grade student at the advanced level in mathematics demonstrates superior performance. He/she gives responses that exhibit advanced understanding of the problem or situation presented. The student consistently demonstrates advanced conceptualization and makes connections within and between the mathematical and real world. The student is able to apply more than one process, use multiple representations, and determine solutions accurately. Reasoning and structure of responses are clearly communicated and justified.

- (a) The student demonstrates these abilities under content standard 1 in the areas of:
 - (i) rational number relationships;
 - (ii) estimation and reasonableness;
 - (iii) number theory;
 - (iv) rational number operations;
 - (v) metric and standard measurement; and
 - (vi) proportional reasoning.
- (b) The student demonstrates these abilities under content standard 2 in the areas of:
 - (i) representing and comparing data;
 - (ii) evaluating data and making conjectures; and

- (iii) finding probability and predicting.
- (c) The student demonstrates these abilities under content standard 3 in the areas of:
 - (i) properties of solids and figures;
 - (ii) congruence and similarity;
 - (iii) transformations including dilations;
 - (iv) angles, surface area, and volume; and
 - (v) justifying relationships.
- (d) The student demonstrates these abilities under content standard 4 in the areas of:
 - (i) representing and generalizing patterns;
 - (ii) linear functions;
 - (iii) multi-step equations and inequalities;
 - (iv) equivalent algebraic expressions; and
 - (v) linear modeling.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE VI GRADE 8 PERFORMANCE DESCRIPTORS AT THE PROFICIENT LEVEL (1) An eighth-grade student at the proficient level in mathematics demonstrates solid academic performance. He/she gives responses that exhibit clear understanding of the problem or situation presented and is able to make connections within the mathematical and/or real world. The student can determine a process, and sufficiently communicate correct problem solving strategies through relevant representations. Although there may be occasional errors, these do not interfere with appropriate strategies. Solutions are reasonable and well-supported.

- (a) The student demonstrates these abilities under content standard 1 in the areas of:
 - (i) rational number relationships;
 - (ii) estimation and reasonableness;
 - (iii) number theory;
 - (iv) rational number operations;
 - (v) metric and standard measurement; and
 - (vi) proportional reasoning.
- (b) The student demonstrates these abilities under content standard 2 in the areas of:
 - (i) representing and comparing data;
 - (ii) evaluating data and making conjectures; and
 - (iii) finding probability and predicting.
- (c) The student demonstrates these abilities under content standard 3 in the areas of:
 - (i) properties of solids and figures;
 - (ii) congruence and similarity;
 - (iii) transformations including dilations;
 - (iv) angles, surface area, and volume; and

- (v) justifying relationships.
- (d) The student demonstrates these abilities under content standard 4 in the areas of:
 - (i) representing and generalizing patterns;
 - (ii) linear functions;
 - (iii) multi-step equations and inequalities;
 - (iv) equivalent algebraic expressions; and
 - (v) linear modeling.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE VII GRADE 8 PERFORMANCE DESCRIPTORS AT THE NEARING PROFICIENCY LEVEL (1) An eighth-grade student at the nearing proficiency level in mathematics demonstrates partial mastery of the prerequisite knowledge and skills fundamental for proficiency. He/she gives responses that exhibit some understanding of the problem or situation presented and can make rudimentary connections between the mathematical and/or real world. The student struggles to communicate effectively and uses limited evidence of representations to show understanding. Although some basic reasoning skills are apparent, computational skills and problem solving strategies are insufficient. Frequent errors and lack of structure detract from mathematical knowledge and skills.

- (a) The student demonstrates these abilities under content standard 1 in the areas of:
 - (i) rational number relationships;
 - (ii) estimation and reasonableness;
 - (iii) number theory;
 - (iv) rational number operations;
 - (v) metric and standard measurement; and
 - (vi) proportional reasoning.
- (b) The student demonstrates these abilities under content standard 2 in the areas of:
 - (i) representing and comparing data;
 - (ii) evaluating data and making conjectures; and
 - (iii) finding probability and predicting.
- (c) The student demonstrates these abilities under content standard 3 in the areas of:
 - (i) properties of solids and figures;
 - (ii) congruence and similarity;
 - (iii) transformations including dilations;
 - (iv) angles, surface area, and volume; and
 - (v) justifying relationships.
- (d) The student demonstrates these abilities under content standard 4 in the areas of:
 - (i) representing and generalizing patterns;
 - (ii) linear functions;
 - (iii) multi-step equations and inequalities;

- (iv) equivalent algebraic expressions; and
- (v) linear modeling.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE VIII GRADE 8 PERFORMANCE DESCRIPTORS AT THE NOVICE LEVEL

(1) An eighth-grade student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental. He/she gives responses that exhibit significant difficulty in understanding basic concepts, reasoning, implementing problem solving strategies, and making connections. Basic skills, representations, structure, and process development are severely lacking even though the student may attempt to solve problems. Substantial errors and lack of communication skills hinder the student's progress.

(a) The student demonstrates these abilities under content standard 1 in the areas of:

- (i) rational number relationships;
- (ii) estimation and reasonableness;
- (iii) number theory;
- (iv) rational number operations;
- (v) metric and standard measurement; and
- (vi) proportional reasoning.

(b) The student demonstrates these abilities under content standard 2 in the areas of:

- (i) representing and comparing data;
- (ii) evaluating data and making conjectures; and
- (iii) finding probability and predicting.

(c) The student demonstrates these abilities under content standard 3 in the areas of:

- (i) properties of solids and figures;
- (ii) congruence and similarity;
- (iii) transformations including dilations;
- (iv) angles, surface area, and volume; and
- (v) justifying relationships.

(d) The student demonstrates these abilities under content standard 4 in the areas of:

- (i) representing and generalizing patterns;
- (ii) linear functions;
- (iii) multi-step equations and inequalities;
- (iv) equivalent algebraic expressions; and
- (v) linear modeling.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE IX UPON GRADUATION PERFORMANCE DESCRIPTORS AT THE ADVANCED LEVEL

(1) A graduating student at the advanced level in

mathematics demonstrates superior performance. He/she gives responses that exhibit advanced understanding of the problem or situation presented. The student consistently demonstrates advanced conceptualization and makes connections within and between the mathematical and real world. The student is able to apply more than one process, use multiple representations, and determine solutions accurately. Reasoning and structure of responses are clearly communicated and justified.

(a) The student demonstrates these abilities under content standard 1 in the areas of:

- (i) quantification;
- (ii) estimation and accuracy;
- (iii) equivalence with multiple notation;
- (iv) properties of numbers and number systems; and
- (v) modeling relationships and change.

(b) The student demonstrates these abilities under content standard 2 in the areas of:

- (i) representing and analyzing data;
- (ii) evaluating validity;
- (iii) rules of probability and expected value;
- (iv) counting methods; and
- (v) curve fitting.

(c) The student demonstrates these abilities under content standard 3 in the areas of:

- (i) conjectures and inductive reasoning;
- (ii) applications of geometric models;
- (iii) multiple geometric approaches;
- (iv) indirect measurement; and
- (v) methods of proof.

(d) The student demonstrates these abilities under content standard 4 in the areas of:

- (i) representing functions;
- (ii) variables and parameters;
- (iii) solving systems of equations and inequalities;
- (iv) families of functions and transformations; and
- (v) analyzing and conjecturing with models.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE X UPON GRADUATION PERFORMANCE DESCRIPTORS AT THE PROFICIENT LEVEL (1) A graduating student at the proficient level in mathematics demonstrates solid academic performance. He/she gives responses that exhibit clear understanding of the problem or situation presented and is able to make connections within the mathematical and/or real world. The student can determine a process and sufficiently communicate correct problem solving strategies through relevant representations. Although there may be occasional errors, these do not interfere with appropriate strategies. Solutions are reasonable and well-

supported.

(a) The student demonstrates these abilities under content standard 1 in the areas of:

- (i) quantification;
- (ii) estimation and accuracy;
- (iii) equivalence with multiple notation;
- (iv) properties of numbers and number systems; and
- (v) modeling relationships and change.

(b) The student demonstrates these abilities under content standard 2 in the areas of:

- (i) representing and analyzing data;
- (ii) evaluating validity;
- (iii) rules of probability and expected value;
- (iv) counting methods; and
- (v) curve fitting.

(c) The student demonstrates these abilities under content standard 3 in the areas of:

- (i) conjectures and inductive reasoning;
- (ii) applications of geometric models;
- (iii) multiple geometric approaches;
- (iv) indirect measurement; and
- (v) methods of proof.

(d) The student demonstrates these abilities under content standard 4 in the areas of:

- (i) representing functions;
- (ii) variables and parameters;
- (iii) solving systems of equations and inequalities;
- (iv) families of functions and transformations; and
- (v) analyzing and conjecturing with models.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE XI UPON GRADUATION PERFORMANCE DESCRIPTORS AT THE NEARING PROFICIENCY LEVEL (1) A graduating student at the nearing proficiency level in mathematics demonstrates partial mastery of the prerequisite knowledge and skills fundamental for proficiency. He/she gives responses that exhibit some understanding of the problem or situation presented and can make rudimentary connections between the mathematical and/or real world. The student struggles to communicate effectively and uses limited evidence of representations to show understanding. Although some basic reasoning skills are apparent, computational skills and problem solving strategies are insufficient. Frequent errors and lack of structure detract from mathematical knowledge and skills.

(a) The student demonstrates these abilities under content standard 1 in the areas of:

- (i) quantification;
- (ii) estimation and accuracy;

- (iii) equivalence with multiple notation;
- (iv) properties of numbers and number systems; and
- (v) modeling relationships and change.
- (b) The student demonstrates these abilities under content standard 2 in the areas of:
 - (i) representing and analyzing data;
 - (ii) evaluating validity;
 - (iii) rules of probability and expected value;
 - (iv) counting methods; and
 - (v) curve fitting.
- (c) The student demonstrates these abilities under content standard 3 in the areas of:
 - (i) conjectures and inductive reasoning;
 - (ii) applications of geometric models;
 - (iii) multiple geometric approaches;
 - (iv) indirect measurement; and
 - (v) methods of proof.
- (d) The student demonstrates these abilities under content standard 4 in the areas of:
 - (i) representing functions;
 - (ii) variables and parameters;
 - (iii) solving systems of equations and inequalities;
 - (iv) families of functions and transformations; and
 - (v) analyzing and conjecturing with models.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE XII UPON GRADUATION PERFORMANCE DESCRIPTORS AT THE NOVICE LEVEL (1) A graduating student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental. He/she gives responses that exhibit significant difficulty in understanding basic concepts, reasoning, implementing problem solving strategies, and making connections. Basic skills, representations, structure, and process development are severely lacking even though the student may attempt to solve problems. Substantial errors and lack of communication skills hinder the student's progress.

- (a) The student demonstrates these abilities under content standard 1 in the areas of:
 - (i) quantification;
 - (ii) estimation and accuracy;
 - (iii) equivalence with multiple notation;
 - (iv) properties of numbers and number systems; and
 - (v) modeling relationships and change.
- (b) The student demonstrates these abilities under content standard 2 in the areas of:
 - (i) representing and analyzing data;
 - (ii) evaluating validity;

- (iii) rules of probability and expected value;
- (iv) counting methods; and
- (v) curve fitting.
- (c) The student demonstrates these abilities under content standard 3 in the areas of:
 - (i) conjectures and inductive reasoning;
 - (ii) applications of geometric models;
 - (iii) multiple geometric approaches;
 - (iv) indirect measurement; and
 - (v) methods of proof.
- (d) The student demonstrates these abilities under content standard 4 in the areas of:
 - (i) representing functions;
 - (ii) variables and parameters;
 - (iii) solving systems of equations and inequalities;
 - (iv) families of functions and transformations; and
 - (v) analyzing and conjecturing with models.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

5. The rules as proposed to be amended provide as follows, new matter underlined, deleted matter interlined:

10.54.4010 MATHEMATICS CONTENT STANDARD 1 (1) To satisfy the requirements of mathematics content standard 1, a student, applying reasoning and ~~must be able to demonstrate that he/she engages in the mathematical processes of~~ problem solving, will use number sense and operations to represent numbers in multiple ways, understand relationships among numbers and number systems, make reasonable estimates, and compute fluently within a variety of relevant cultural contexts and reasoning, estimation, communication, connections and applications, ~~and uses appropriate technology.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4011 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 1 FOR THE END OF GRADE 4 (1) The benchmark for mathematics content standard 1 for a student at the end of grade 4 is the ability to:

- (a) demonstrate relationships among whole numbers; identify place value up to 100,000, and compare numbers (e.g., greater than, less than, and equal to);
- (b) estimate sums, differences, products, and quotients when solving problems; add, subtract, multiply (three-digit by two-digit factors), and divide (two-digit dividends by one-digit divisors) to solve problems; and demonstrate fluency with basic facts;
- (c) develop multiplication and division concepts, apply number and operation models and strategies, and reason and justify using properties of operations;

(d) identify and model common fractions such as, tenths, fourths, thirds, and halves and decimals such as, money and place value to 0.001, and recognize and compare equivalent representations; and

(e) select and apply appropriate standard units and tools to measure length, time, and temperature within relevant scientific and cultural situations.

~~(a) solve problems from many contexts using a variety of strategies (e.g., estimate, make a table, look for a pattern, simplify the problem) and explain the methods for solving these problems;~~

~~(b) apply estimation strategies throughout the problem-solving process;~~

~~(c) communicate mathematical ideas in a variety of ways (e.g., written, verbal, concrete, pictorial, graphical, algebraic);~~

~~(d) recognize and investigate the relevance and usefulness of mathematics through applications, both in and out of school; and~~

~~(e) select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, and computer.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4012 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 1 FOR THE END OF GRADE 8 (1) The benchmark for mathematics content standard 1 for a student at the end of grade 8 is the ability to:

(a) recognize, model, and compare different forms of integers and rational numbers including percents, fractions, decimals, and numbers using exponents and scientific notation;

(b) select and apply appropriate estimation strategies to judge the reasonableness of solutions to problems including those computed on a calculator; and demonstrate correct use of order of operations;

(c) use number theory concepts such as prime factorization, greatest common factor, and least common multiple in problem situations;

(d) compute fluently and solve multi-step problems using integers, fractions, decimals, and numbers in exponential form;

(e) use metric and standard units of measurement in relevant scientific and cultural situations, compare and convert within systems, and use appropriate technology; and

(f) understand and apply proportional relationships to model real world situations and to solve problems involving rates, ratios, proportions, percents, and direct variation.

~~(a) formulate and solve multi-step and nonroutine problems using a variety of strategies and generalize methods to new problem situations;~~

~~(b) select and apply appropriate estimation strategies throughout the problem-solving process;~~

~~(c) interpret and communicate mathematical ideas and logical arguments using correct mathematical terms and notations;~~

~~(d) recognize and investigate the relevance and usefulness of mathematics through applications, both in and out of school; and~~

~~(e) select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, computer, and data collection devices.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4013 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 1 UPON GRADUATION (1) The benchmark for mathematics content standard 1 for a student upon graduation is the ability to:

(a) use multiple notations to perform and interpret the effects of operations on very large and very small numbers with and without technology;

(b) identify situations where estimation is appropriate and determine the degree of accuracy needed for a given problem situation (and the appropriate precision in which to report answers);

(c) given a representation of a number or expression, find equivalent representations using multiple notations (e.g., x to the $\frac{1}{2}$ vs. the square root of x and visual representation of multiplying binomials);

(d) analyze and apply the properties of numbers and number systems; and

(e) identify givens and unknowns in familiar and unfamiliar situations (e.g., finance, culture, and nature) and describe relationships between variables.

~~(a) recognize and formulate problems from situations within and outside mathematics and apply solution strategies to those problems;~~

~~(b) select, apply, and evaluate appropriate estimation strategies throughout the problem-solving process;~~

~~(c) formulate definitions, make and justify inferences, express generalizations, and communicate mathematical ideas and relationships;~~

~~(d) apply and translate among different representations of the same problem situation or of the same mathematical concept;~~

~~(e) model connections between problem situations that arise in disciplines other than mathematics; and~~

~~(f) select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, computer, and data collection devices.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4020 MATHEMATICS CONTENT STANDARD 2 (1) To satisfy the requirements of mathematics content standard 2, a student, applying reasoning and problem solving, will use data representation and analysis, simulations, probability statistics, and statistical methods to evaluate information and make informed decisions within a variety of relevant cultural contexts. ~~must be able to demonstrate that he/she has an understanding of and an ability to use numbers and operations.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4021 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 2 FOR END OF GRADE 4 (1) The benchmark for mathematics content standard 2 for a student at the end of grade 4 is the ability to:

(a) collect, represent, and organize data in tables, dot plots, bar graphs, pictographs, and stem and leaf plots using technology when appropriate;

(b) solve problems and make decisions using data descriptors such as minimum, maximum, median, and mode within scientific and cultural contexts when relevant; and

(c) describe events from multicultural contexts as likely or unlikely and discuss the degree of likelihood using words such as certain, equally likely, and impossible.

~~(a) exhibit connections between the concrete and symbolic representation of a problem or concept;~~

~~(b) use the number system by counting, grouping, and applying place value concepts;~~

~~(c) model, explain, and use basic facts, the operations of addition and subtraction of whole numbers, and mental mathematics;~~

~~(d) model and explain multiplication and division of whole numbers; and~~

~~(e) model and explain part/whole relationships in everyday situations.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4022 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 2 FOR END OF GRADE 8 (1) The benchmark for mathematics content standard 2 for a student at the end of grade 8 is the ability to:

(a) collect data from a variety of contexts (e.g., science, history, and culture); and organize and represent data in box plots, scatter plots, histograms, and circle graphs using technology when appropriate;

(b) interpret, analyze, and evaluate data using mean, median, range, and quartiles to identify trends and make decisions and predictions about data within scientific and cultural contexts when relevant; and

(c) create sample spaces and simulations from events found in different cultures, determine experimental and theoretical probabilities, and use probability to make predictions.

~~(a) use the four basic operations with whole numbers, fractions, decimals, and integers;~~

~~(b) use mental mathematics and number sense in using order of operations and order relations for whole numbers, fractions, decimals, and integers;~~

~~(c) use the relationships and applications of ratio, proportion, percent, and scientific notation; and~~

~~(d) develop and apply number theory concepts (e.g., primes, factors, multiples) in real world and mathematical problem situations.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4023 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 2 UPON GRADUATION (1) The benchmark for mathematics content standard 2 for a student upon graduation is the ability to:

(a) select, create, and compare graphical or numerical representations of data sets using technology when appropriate and reason about distributions using measures of central tendency and spread (e.g., percentiles, quartiles, inter-quartile range, and standard deviation);

(b) evaluate the validity of reports based on collected and/or published data by considering the source of the data, the design of the study, and the way data are displayed, analyzed, and interpreted;

(c) make, evaluate, and justify decisions based on probabilities in multicultural problem situations (e.g., finding expected value and using rules of probability);

(d) use technology as needed to determine the possible number of outcomes for an event or compound event using the fundamental counting principle, permutations, combinations, and other systematic counting methods; and

(e) model two-variable data using curve fitting with and without technology and write an equation for a given model and decide when or if predictions based on this equation are valid.

~~(a) use and understand the real number system, its operations, notations, and the various subsystems; and~~

~~(b) use definitions and basic operations of the complex number system.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4030 MATHEMATICS CONTENT STANDARD 3 (1) To satisfy the requirements of mathematics content standard 3, a student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant cultural contexts. ~~must be able to demonstrate that he/she uses algebraic concepts, processes, and language to model and solve a variety of real world and mathematical problems.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4031 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 3 FOR END OF GRADE 4 (1) The benchmark for mathematics content standard 3 for a student at the end of grade 4 is the ability to:

(a) describe, compare, and analyze attributes of two-dimensional shapes;

(b) describe attributes of three-dimensional shapes such as cubes and other rectangular prisms, pyramids, cylinders, cones, and spheres;

(c) use spatial reasoning to identify slides and flips of congruent figures within cultural and artistic contexts;

(d) estimate and measure linear attributes of objects in metric units such as centimeters and meters and customary units such as inch, foot, and yard; and

(e) define and determine area and perimeter of common polygons using concrete tools such as grid paper, objects, or technology and justify the strategy used.

~~(a) use symbols (e.g., boxes or letters) to represent numbers in simple situations;~~

~~(b) explore the use of variables and open sentences to express relationships (e.g., missing addend); and~~

~~(c) use inverse operations and other strategies to solve number sentences.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4032 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 3 FOR END OF GRADE 8 (1) The benchmark for mathematics content standard 3 for a student at the end of grade 8 is the ability to:

(a) define, classify, and compare properties of solids and plane figures, including lines and angles;

(b) use spatial reasoning to determine congruence, similarity, and symmetry of objects in mathematics, art, science, and culture;

(c) define, identify, and execute transformations including translations, rotations, reflections, and dilations with appropriate technology;

(d) measure and compute angles, perimeter, area, surface area, and volume including the use of formulas and choosing appropriate units; and

(e) develop informal arguments to verify geometric relationships and solve problems such as an informal justification of the Pythagorean Theorem in a variety of contexts.

~~(a) understand the concepts of variable, expression, and equation;~~

~~(b) represent situations and number patterns using tables, graphs, verbal rules, equations, and models;~~

~~(c) recognize and use the general properties of operations (e.g., the distributive property);~~

~~(d) solve linear equations using concrete, numerical, and algebraic methods; and~~

~~(e) investigate inequalities and nonlinear relationships informally.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4033 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 3 UPON GRADUATION (1) The benchmark for mathematics content standard 3 for a student upon graduation is the ability to:

(a) formulate and evaluate conjectures about geometric objects and their properties, with and without technology, applying inductive reasoning when appropriate;

(b) use spatial reasoning and geometric models to solve problems with and without technology in the contexts of art, science, and culture;

(c) identify, analyze, and use transformational, coordinate, and synthetic

geometric approaches to solve problems;

(d) determine measures of two- and three-dimensional objects and their elements using trigonometric ratios, proportionality, the Pythagorean Theorem, and angle relationships; and

(e) establish the validity of geometric conjectures, using deductive reasoning, indirect proof, and counterexamples, and critique arguments made by others.

~~(a) use algebra to represent patterns of change;~~

~~(b) use basic operations with algebraic expressions;~~

~~(c) solve algebraic equations and inequalities: linear, quadratic, exponential, logarithmic, and power;~~

~~(d) solve systems of algebraic equations and inequalities, including use of matrices; and~~

~~(e) use algebraic models to solve mathematical and real world problems.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4040 MATHEMATICS CONTENT STANDARD 4 (1) To satisfy the requirements of mathematics content standard 4, a student, applying reasoning and problem solving, will use algebraic concepts and procedures to understand processes involving number, operation, and variables and will use procedures and function concepts to model the quantitative and functional relationships that describe change within a variety of relevant cultural contexts. must be able to demonstrate that he/she has an understanding of shape and an ability to use geometry.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4041 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 4 FOR END OF GRADE 4 (1) The benchmark for mathematics content standard 4 for a student at the end of grade 4 is the ability to:

(a) describe, extend, and make generalizations about geometric or numeric patterns;

(b) use letters, boxes, or symbols to represent numbers in simple expressions or equations to demonstrate a basic understanding of variables;

(c) use number patterns to investigate properties of numbers such as even or odd and properties of operations such as commutative, associative, distributive, and the multiplicative and additive identities;

(d) develop an understanding of equivalence by expressing numbers, measures, and numerical expressions involving operations in a variety of ways; and

(e) model problem situations with manipulatives or technology and use multiple representations such as words, pictures, tables, or graphs to draw conclusions using cultural contexts when relevant.

~~(a) describe, model, and classify two and three dimensional shapes;~~

~~(b) investigate and predict results of combining, subdividing, and changing shapes; and~~

~~(c) identify lines of symmetry, congruent and similar shapes, and positional~~

relationships.

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4042 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 4 FOR END OF GRADE 8 (1) The benchmark for mathematics content standard 4 for a student at the end of grade 8 is the ability to:

(a) create and use tables, graphs or diagrams, symbolic expressions, and verbal descriptions to represent, analyze, and generalize a variety of patterns involving numbers and operations;

(b) identify linear and non-linear functional relationships and contract their properties using tables, graphs, or equations with appropriate technology;

(c) use number properties and inverse operations to solve multi-step equations and inequalities involving a single variable;

(d) recognize, simplify, and generate equivalent forms of algebraic expressions, justifying each step with properties of operations; and

(e) identify and compute rate of change/slope and intercepts from equations, graphs, and tables; model and solve contextual problems involving linear proportions or direct variation using cultural contexts when relevant.

~~(a) identify, describe, construct, and compare plane and solid geometric figures;~~

~~(b) understand and apply geometric properties and relationships (e.g., the Pythagorean theorem);~~

~~(c) represent geometric figures on a coordinate grid;~~

~~(d) explore properties and transformations of geometric figures; and~~

~~(e) use geometry as a means of describing the physical world.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4043 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 4 UPON GRADUATION (1) The benchmark for mathematics content standard 4 for a student upon graduation is the ability to:

(a) represent functions in a variety of ways including tables, graphs or diagrams, verbal descriptions, and symbolic expressions in recursive and explicit form and justify the choice of an appropriate form for solving a given problem;

(b) determine the appropriate symbolic representation of a given contextual situation (e.g., variables and parameters in equations, inequalities, functions, and matrices);

(c) solve a variety of equations, inequalities and systems of equations and inequalities, justify the solution process, and interpret the solution in context;

(d) analyze the effects of transformations on families of functions and recognize their characteristics and represent and use functions in equivalent forms to identify and perform transformations; and

(e) given data or a problem situation, select and use an appropriate function model to analyze results or make a prediction with and without technology using

cultural contexts when relevant.

- ~~(a) construct, interpret, and draw three-dimensional objects;~~
- ~~(b) classify figures in terms of congruence and similarity and apply these relationships;~~
- ~~(c) translate between synthetic and coordinate representations;~~
- ~~(d) deduce properties of figures using transformations, coordinates, and vectors in problem solving; and~~
- ~~(e) apply trigonometric ratios (sine, cosine, and tangent) to problem situations involving triangles.~~

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

6. The board proposes to repeal the following rules:

10.54.4050 MATHEMATICS CONTENT STANDARD 5 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4051 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 5 FOR END OF GRADE 4 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4052 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 5 FOR END OF GRADE 8 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4053 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 5 UPON GRADUATION AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4060 MATHEMATICS CONTENT STANDARD 6 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4061 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 6 FOR END OF GRADE 4 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4062 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 6 FOR END OF GRADE 8 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4063 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 6 UPON GRADUATION AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4070 MATHEMATICS CONTENT STANDARD 7 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4071 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 7 FOR END OF GRADE 4 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4072 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 7 FOR END OF GRADE 8 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4073 BENCHMARK FOR MATHEMATICS CONTENT STANDARD 7 UPON GRADUATION AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4087 ADVANCED MATHEMATICS PERFORMANCE STANDARDS FOR END OF GRADE 4 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4088 PROFICIENT MATHEMATICS PERFORMANCE STANDARDS FOR END OF GRADE 4 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4089 NEARING PROFICIENCY MATHEMATICS PERFORMANCE STANDARDS FOR END OF GRADE 4 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4090 NOVICE MATHEMATICS PERFORMANCE STANDARDS FOR END OF GRADE 4 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4091 ADVANCED MATHEMATICS PERFORMANCE STANDARDS FOR END OF GRADE 8 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4092 PROFICIENT MATHEMATICS PERFORMANCE STANDARDS FOR END OF GRADE 8 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4093 NEARING PROFICIENCY MATHEMATICS PERFORMANCE STANDARDS FOR END OF GRADE 8 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4094 NOVICE MATHEMATICS PERFORMANCE STANDARDS FOR END OF GRADE 8 AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4095 ADVANCED MATHEMATICS PERFORMANCE STANDARDS UPON GRADUATION AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4096 PROFICIENT MATHEMATICS PERFORMANCE STANDARDS UPON GRADUATION AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4097 NEARING PROFICIENCY MATHEMATICS PERFORMANCE STANDARDS UPON GRADUATION AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.54.4098 NOVICE MATHEMATICS PERFORMANCE STANDARDS UPON GRADUATION AUTH: 20-2-114, MCA; IMP: 20-2-121, 20-3-106, 20-7-101, MCA

7. Concerned persons may submit their data, views, or arguments either orally or in writing at the hearing. Written data, views, or arguments may also be submitted to: Steve Meloy, P.O. Box 200601, Helena, MT 59620-0601, telephone: (406) 444-6576, FAX: (406) 444-0847, e-mail: smeloy@mt.gov and must be received no later than 5:00 p.m., June 25, 2009.

8. Steve Meloy, Executive Secretary of the Board of Public Education has been designated to preside over and conduct this hearing.

9. The board maintains a list of interested persons who wish to receive notices of rulemaking actions proposed by this board. Persons who wish to have their name added to the list shall make a written request that includes the name, e-mail, and mailing address of the person to receive notices and specifies for which program the person wishes to receive notices. Notices will be sent by e-mail unless a mailing preference is noted in the request. Such written request may be mailed or delivered to the contact person in 7 above or may be made by completing a request form at any rules hearing held by the board.

10. An electronic copy of this Proposal Notice is available through the Secretary of State's web site at <http://sos.mt.gov/ARM/Register>. The Secretary of State strives to make the electronic copy of the Notice conform to the official version of the Notice, as printed in the Montana Administrative Register, but advises all concerned persons that in the event of a discrepancy between the official printed text of the Notice and the electronic version of the Notice, only the official printed text will be considered. In addition, although the Secretary of State works to keep its web site accessible at all times, concerned persons should be aware that the web site may be unavailable during some periods, due to system maintenance or technical problems.

11. The bill sponsor contact requirements of 2-4-302, MCA, do not apply.

/s/ Steve Meloy
Steve Meloy
Rule Reviewer

/s/ Patty Myers
Patty Myers, Chairperson
Board of Public Education

Certified to the Secretary of State May 18, 2009.